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Capacity Constrained Supply Function Equilibria: Modeling and Simulations

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Abstract

This paper models the restructured wholesale electricity markets as oligopolies facing uncertain demand and "capacity constraints" where each firm chooses as its strategy a "supply function", as introduced by Klemperer and Meyer. To facilitate the computation of optimal strategies, the paper considers piecewise linear bid (or supply) functions, imposing restrictions to allow for just a few knots. It then suggests algorithms to find equilibrium strategies of firms and implements simulations consisting of various combinations of firms. The simulation results show that the firms' equilibrium supply-functions are steeper with fewer firms, with a smaller variance of demand, and with more severe asymmetry between small firms and a large firm. This paper also compares supply function equilibrium with capacity constraints to the Cournot equilibrium assuming storability and Cournot points of every demand realization. The results show that the Lerner index of the supply function equilibrium with capacity constraints is considerably smaller than that of the Cournot equilibrium assuming storability and Cournot points. The Lerner index decreases as the number of firms rises and increases as the asymmetry between firm sizes grows.

Key Words : supply-function equilibrium, electricity, capacity constraints
JEL Codes : L1, Q3

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Ⅰ. Introduction

The proliferation of the wholesale electricity market in various jurisdictions throughout the world is a salient feature in the restructuring of the electricity business[1]. Various electricity markets have been implemented in which the principal competitive mechanism is an auction.

The most promising approach for the analysis of these new electricity markets is based on the idea of supply function equilibria introduced by Klemperer and Meyer(1989). This approach was first applied to the electricity market by Green and Newbery(1992), who viewed that the set-up of supply function equilibrium fit well with the structure of the electricity markets. Klemperer and Meyer show that the supply function equilibrium is characterized by differential equations. In general, the supply function equilibrium approach yields multiple equilibria; for any given set of supply and demand conditions, the market price and vector of firm outputs are not uniquely specified, and many researches have focused on the range and uniqueness of the supply function equilibrium. The range of equilibria can be limited by capacity (Green and Newbery, 1992; Baldick and Hogan, 2002). Genc and Reynolds(2004) analyze how pivotal producers reduce the range of equilibria. From an analytical perspective, Holmberg(2008) shows that there is a unique equilibrium when there are symmetric producers with strictly convex cost functions, and Holmberg(2007) shows that there is a unique equilibrium when producers have identical constant marginal costs. In reality, however, the marginal cost function for firms is usually not constant and their production capacities are

1) See Gross, G(1994)

asymmetric. Because Klemperer & Meyer's first-order conditions constitute a system of non-autonomous ordinary differential equations, solving this system analytically is difficult. Baldick and Hogan(2002) posit that solutions that meet the requirement that supply functions must be non-decreasing are difficult to find. There are three exceptions to this assertion, including symmetric firms with identical cost functions, cases with affine marginal costs and no capacity constraints, and cases in which variations in demand are small. However, these situations are too limited to represent real market structure. Due to the multiplicity of supply function equilibrium(SFE) and the difficulty in handling non-autonomous ordinary differential equality, predicting outcomes and generating comparative statics from the SFE model is difficult, especially when realistic features of the electricity market structure are taken account such as asymmetric firm capacities and increasing marginal cost and price dependent demand.

In practice, this problem may be avoided at the cost of limiting the strategic spaces of firms; for example, certain applications limit firms to linear supply functions2). This paper adopts an approach that is used more often in operations research than by academic economists. Instead of simplifying the model to make the computation of optimal actions possible, more details may be incorporated into the model with a simplified solution concept by limiting the set of admissible decision alternatives. Here a restriction is imposed to permit only piecewise linear bidding functions instead of allowing suppliers in a power exchange to submit any type of bid function. Such a restriction is not as limiting as it may first appear because (i) in the real world many bidders use rules of thumb, such as markup or linear bidding strategies, instead of maximally optimal bidding strategies and (ii) there may be institutional restrictions that prevent bidders from adopting an optimal bidding rule even if they desire to do so. For example, the rules of the England and Wales Power Pool limit the bid functions a supplier can submit to

2) Recent applications of this methodology include Baldick et al.(2004).

piecewise linear functions with up to three break-points. This approach is particularly effective in incorporating the unique features and operational aspects of electricity generation, consumption, and exchange mechanisms.

This paper's model has more realistic representations, including: (i) the explicit incorporation of the firms' capacity constraints, and (ii) price-sensitive demand described by a random variable.

The principal thrust of this work involves the application of the model to evaluate the performance of the electricity exchange as a function of several important factors, such as the number of firms, the difference in firm sizes and the variability of demand. The paper suggests algorithms to find equilibrium strategies for firms, and it implements simulations and compares the performance of various market structures.

Ⅱ. Model

Let us consider a market with $i = 1, 2, \ldots, N$ firms. Each firm owns and operates a single generation unit with capacity K_i, and we use q_i to denote the output of the unit of firm i. The output q_i is a function of the price and N firms compete for the right to serve the demand through a sealed-bid, uniform price auction. Let the demand be a monotonically nonincreasing function of price. Such a relationship captures the reduction in customer demand as prices increase. The incorporation of the price sensitivity of demand makes the equilibrium price and quantity interdependent. Demand is also considered as a random variable determined by a random factor ϵ. It follows that the equilibrium price and the allocation of the equilibrium quantity are also random variables.

Each firm submits a supply curve and the aggregated N supply curves form the market supply curve. The market equilibrium is, therefore, determined as follows:

$$D(p,\epsilon) = \sum_{i=1}^{N} q_i \tag{1}$$

The equilibrium price p is determined by the supply demand condition in eq.(1).

The production costs of a unit are expressed as a polynomial, or in certain cases, as piecewise polynomial function of the unit output. In this exposition, a piecewise quadratic representation is adopted. $c_{i,m}(q_i)$ is used to denote the costs of unit i to produce q_i where $q_{i,m} \le q_i \le q_{i,\,m+1}$ and m is the index of intervals of the piecewise polynomial function.

$$c_{i,m}(q_i) = \sum_{\eta=0}^{2} c_{i,\eta,m}[q_i]^{\eta},\ q_{i,m} \le q_i \le q_{i,m+1} \tag{2}$$

$$c_{i,m}(q_i) = \infty \qquad K_i \le q_i \tag{3}$$

The polynomial coefficients $c_{i,\eta,\cdot}$, $\eta = 0, 1, 2$ are selected to reflect the so-called "no load" costs of the unit and to ensure that the marginal cost is a nondecreasing function of output. Marginal cost has a simple piecewise linear form, expressed as follows:

$$c'_{i,m}(q_i) = 2 * c_{i,2,m} q_i + c_{i,1,m}, \qquad q_{i,m} \le q_i \le q_{i,\,m+1} \tag{4}$$

$$c'_{i,m}(q_i) = \infty, \qquad K_i \le q_i \tag{5}$$

The form in (4) and (5) is used to specify the form of the bid function for each unit i. For simplicity, piecewise linear bid functions are considered. Let the bid function in a unit's output range be expressed as follows:

$$b_{i,m}(q_i) = b_{i,2,m}q_i + b_{i,1,m}, \qquad q_{i,m} \le q_i \le q_{i,m+1} \tag{6}$$
$$b_{i,m}(q_i) = \infty, \qquad K_i \le q_i \tag{7}$$

Here, $b_i(q_i)$ is the bid price of the unit i for supplying q_i. Because price and cost are independent, $b_i(q_i)$ does not need to have any relationship with the marginal cost. The parameters $b_{i,1,m}$ and $b_{i,2,m}$ are selected to ensure that the inverse function $b_i^{-1}(\bullet)$ exists, and $\beta_i \doteq \{b_{i,1,m}, b_{i,2,m}\}$ specifies the bidding strategy of firm i. The supply function $q_i^s(p, \beta_i)$ is formally defined to be a function of the price p and the bidding strategy β_i and represented by the following:

$$q_i = q_i^S(p, \beta_i) = \begin{cases} K_i & p^{\max} \ge p > b_i(K_i) \\ b_i^{-1}(p) & p \le b_i(K_i) \end{cases} \tag{8}$$

The term $p^{\max}$ is a specified maximum price and K_i is the maximum output of unit i.

Next, the market supply curve is constructed. Let $\beta \doteq \{\beta_1, \beta_{2,} \cdots \beta_N\}$ be the collection of N firm's supply functions, resulting in the market supply function when N firms' supply functions are aggregated:

$$q(p,\beta) = \sum_{i=1}^{N} q_i^s(p, \beta_i) \tag{9}$$

The equilibrium condition in eq (1) is set as follows:

$$D(p,\epsilon) = q(p,\beta) \tag{10}$$

and determines the equilibrium price p. Clearly, p is a function of ϵ and the collection of strategies β.

The profit of firm i is given in the following

$$\Pi_i(\beta_i, p) = p^* q_i - c_i(q_i) = p q_i^s(p, \beta_i) - c_i(q_i) \tag{11}$$

Each firm can determine its strategy β_i to maximize its profits. However, the equilibrium price p is a function of β_i and β_{-i}, where, β_{-i} is the complement of β_i in the set β.

The strategy set β is made up of the strategies β_i that are selected to

maximize the expected value of the profits. Thus, each firm i would choose β_i to maximize the following:

$$E\{\Pi_i(\beta_i, p)\} = E\{p^* q_i - c_i(q_i)\} = E\{p q_i^s(p, \beta_i) - c_i(q_i)\} \tag{12}$$

Ⅲ. Description of algorithms

This section describes algorithms to find equilibrium supply function for each firm through simulation.

By definition of supply function equilibrium, each firm must select the best supply function under the restrictions imposed by this paper, given the other firms' supply functions. Thus, the supply function of a firm must make the biggest expected profit among other alternatives that satisfy our restrictions. The actual simulation process repeats to find every firm's supply function that maximizes expected profit and to update the firm's strategy with the supply function of each firm until all firms' supply function converges. When searching for a firm's optimal supply function, the strategies of the other firms are fixed. Because demand is uncertain, profits expected from random draws must be calculated. Due to the restrictions imposed on supply functions, the optimal supply function of a firm is a vector of an intercept and slopes which maximize expected profit. ‘Fmin’ function in MATLAB is used to find the optimal supply functions.

Algorithm 1 is for the calculation of the expected profit with a strategy of a specific firm, given the strategies of the other firms. The function obtained from Algorithm 1 is an objective function that aims to maximize at Algorithm 2 with ‘fmin’ function. Algorithm 2 is for the finding of the supply function equilibrium.

Algorithm 1: Given the other firms' (supply function) strategies, calculate the expected profit with a strategy of a specific firm:

```
Initialization
For 1:number of iteration
  Draw demand from a specific probability distribution
  Initialize price
  While demand is not equal to supply
    Calculate demand and supply at the price
    If demand is greater than aggregate supply, increase the price
    If aggregate supply is greater than demand, decrease the price
  End
  Find quantity corresponding to equilibrium price
  Calculate profit
End
Calculate the expected profit
```

Algorithm 2 : find an equilibrium strategy

```
Initialization
While at least one firm’s strategy not converged
```

```
  For 1:number of firms
    Compute a firm’s optimal response based on the other firms’ strategies
     (find the strategy which maximize the expected profit from
     Algorithm 1 using ‘fmin’ function in MATLAB)
    Update the firm’s strategy
  End
End
```

Ⅳ. Simulation Results

To compare the performance of various market structures, simulations are implemented that consist of various combinations of firms. Marginal cost(MC) curves are considered; these are piecewise linear and continuous functions, with one kink point. Each firm has its own capacity so that marginal cost beyond each firm's capacity is infinite. This paper supposes market capacity to be 30, and in symmetric market structures, the capacity of each firm is assumed to be 30 divided by the number of firms. In asymmetric market structures, a large firm is either two or three times larger than the small firm(s), but the summation of all firms' capacity remains at 30. The marginal cost curves of symmetric firms are identical. The MC curve of a large firm that is twice as large as that of a small firm is exactly the horizontal summation of two identical small firms, and the MC curve of a large firm three times as large as a small firm is exactly the horizontal summation of three identical small firms. If the horizontal summation of MC curves of all firms in the market is determined, then the MC curve of the market is expressed as follows:

$$c'(q) = 2 + 0.25q, \qquad 0 \leq q \leq 15 \qquad (13)$$

$$c'(q) = 2 + 0.25(15) + 0.5q, \qquad 15 \leq q \leq 30 \qquad (14)$$

$$c'(q) = \infty, \qquad 30 \leq q \qquad (15)$$

To facilitate computations, piecewise linear and continuous bid(or supply) functions are considered, and restrictions are imposed to allow for only one knot. Another restriction imposed by this paper is to place the kink point of two slopes at the same kink point as that of the marginal cost curve, meaning that the optimal strategy of each firm consists of a intercept and two slopes.

Let demand be $D(p, \epsilon) = 2 - 1*p + \epsilon$, where ϵ is a random factor drawn from a certain distribution. When using a 'uniform distribution' for ϵ, ϵ is drawn from an uniform distribution where the domain is [0, 40].

First, to compare the performance of market structures with different numbers of firms, 5 simulations are performed with the equal-sized firm structures of 2, 3, 4, 5 and 10 firms, respectively, where the random factor of the demand is drawn from a uniform distribution. Table 1 provides the simulation results for the optimal strategies of firms with the aforementioned restrictions.

Table 1. Equilibrium supply curve of each firm of uniform distribution, symmetric firms

number of firms	intercept	the first slope	the second slope
2	1.8324 (2)*	1.0888 (0.50)	1.8741 (1.00)
3	1.8702 (2)	1.2044 (0.75)	2.2336 (1.50)
4	1.9006 (2)	1.3985 (1.00)	2.6556 (2.00)
5	1.9211 (2)	1.6180 (1.25)	3.1126 (2.50)
10	1.9629 (2)	2.8139 (2.50)	5.5364 (5.00)

* intercepts and slopes of MC curves are in parentheses.

To compare market performance, these supply functions must be aggregated. Table 2 offers aggregated supply curves corresponding to each market structure.

The results of the simulation are intuitive, as aggregated supply functions are steeper with fewer firms. In the market composed of 10 firms, the firm's optimal strategy is very close to the marginal cost pricing.

Table 2. Aggregate supply curve of uniform distribution, symmetric firms case

number of firms	intercept	the first slope	the second slope
marginal cost pricing	2	0.25	0.5
2	1.83	0.545	0.935
3	1.87	0.4016	0.743
4	1.90	0.35	0.665
5	1.92	0.324	0.622
10	1.96	0.28	0.55374

Second, to see the effects of demand variances on the optimal strategies of firms, simulations are carried out with different demand distributions, beta distributions where the mean and range are the same as the uniform distribution considered earlier but where the variances differ from the uniform distribution. The parameters of beta distribution 1 are(2.5, 2.5) and the parameters of beta distribution 2 are(5.5, 5.5). The mean of the uniform distribution and the two beta distributions are the same, but the variance of beta distribution 2 is one fourth of the uniform distribution and one half of that of beta distribution 1. The variance of beta distribution 1 is smaller than the uniform distribution but greater than that of beta distribution 2. Table 3 provides the simulation results for the optimal strategies of firms and the aggregate supply functions of beta distribution 1 and Table 4 provides the simulation results for the optimal strategies of firms and the aggregate supply functions of beta distribution 2.

Table 3. Equilibrium supply curve of each firm and aggregate supply function of beta distribution 1, symmetric firms case

Equilibrium supply curve of each firm			
number of firms	intercept	the first slope	the second slope
2	1.6860(2)*	1.1110(0.50)	2.0735(1.00)
3	1.7307(2)	1.2406(0.75)	2.3368(1.50)
4	1.7988(2)	1.4301(1.00)	2.7460(2.00)
5	1.8425(2)	1.6467(1.25)	3.1951(2.50)
10	1.9237(2)	2.8419(2.50)	5.5967(5.00)
Aggregate supply curve			
number of firms	intercept	the first slope	the second slope
marginal cost pricing	2	0.25	0.5
2	1.6860	0.5555	1.0368
3	1.7307	0.4135	0.7789
4	1.7988	0.3575	0.6865
5	1.8425	0.3293	0.6390
10	1.9237	0.2842	0.5597

* intercepts and slopes of MC curves are in parentheses.

Table 4. Equilibrium supply curve of each firm and aggregate supply curve of beta distribution 2, symmetric firms

Equilibrium supply curve			
number of firms	intercept	the first slope	the second slope
2	1.5353(2)	1.1303(0.50)	2.8714(1.00)
3	1.5280(2)	1.2911(0.75)	2.5441(1.50)
4	1.6396(2)	1.4813(1.00)	2.8865(2.00)
5	1.7138(2)	1.6974(1.25)	3.3088(2.50)
10	1.8554(2)	2.8943(2.5)	5.6690(5.00)
Aggregate supply curve			
number of firms	intercept	the first slope	the second slope
marginal cost pricing	2	0.25	0.5
2	1.5353	0.5652	1.4357
3	1.5280	0.4304	0.8480
4	1.6396	0.3703	0.7216
5	1.7138	0.3395	0.6618
10	1.8554	0.2894	0.5669

* intercepts and slopes of MC curves are in parenthesis.

Table 5 shows a comparison of the supply functions that results from the simulations conducted with different demand distributions. The supply curves are steeper when demand is drawn from the beta distributions than when demand is drawn from the uniform distribution. Additionally, the supply curve is steeper when demand is drawn from beta distribution 1 than when it is drawn from beta distribution 2. The variance of beta distribution 1 is smaller than that of the uniform distribution but greater than that of beta distribution 2, and the equilibrium supply functions of beta distribution 1 is steeper than those of the uniform distribution, but it is gentler than those of beta distribution 2.

Table 5. Comparison of supply functions equilibrium of different demand distributions

demand distribution	intercept	the first slope	the second slope
uniform distribution 2 firms	1.8324	1.0888	1.8741
beta distribution1 2 firms	1.6860	1.1110	2.0735
beta distribution2 2 firms	1.5353	1.1303	2.8714
uniform distribution 3 firms	1.8702	1.2044	2.2336
beta distribution1 3 firms	1.7307	1.2406	2.3368
beta distribution2 3 firms	1.5280	1.2911	2.5441

So far, we have addressed a market structure that, is composed of symmetrically sized firms with identical marginal cost curves. Next, we address market structures composed of different sized of firms. Third, to compare the performance of the market structure composed of the different sizes of firms, 3 simulations are conducted with 2, 3 and 4 different sized firms, respectively, where the random factor of the demand is drawn from a uniform distribution. In such asymmetric market structures, there is one large firm and one or more small firm of the same size. Table 6 shows the simulation results regarding a market structure in which there is one large firm which is twice as large as the small firms.

Table 7 is a presentation of markets where the large firm is three times the size of the small firms. Table 6 and 7 show that small firms submit their supply functions more aggressively than do large firms. The difference between the supply function curves and the MC curves of small firms is smaller than such difference in one large firm. For example, in a market structure of two firms in which a large firm is twice as large as the small one, the second slope of the small firm is approximately 1.5 times the size of the MC curve while the second slope of the large firm is approximately 2 times the size of the MC curve.

Table 6. Supply functions where the large firm is twice as large as other firms

Market structure: one large firm and one small firm			
	intercept	the first slope	the second slope
SF of small firm	2.0332(2)*	1.2527(0.75)	2.2049(1.5)
SF of large firm	1.5808(2)	1.0967(0.375)	1.4689(0.75)
Market structure: one large firm and two small firms			
	intercept	the first slope	the second slope
SF of small firms	1.9565(2)	1.4185(1)	2.6522(2)
SF of large firm	1.6013(2)	1.1016(0.5)	1.6324(1)
Market structure: one large firm and three small firms			
	intercept	the first slope	the second slope
SF of small firms	1.9477(2)	1.6267(1.25)	3.1207(2.5)
SF of large firm	1.6793(2)	1.1525(0.625)	1.8502(1.25)

* intercepts and slopes of MC curves are in parentheses

Table 7. Supply functions where the large firm is three times as large as other firms

Market structure: one large firm and one small firm			
	intercept	the first slope	the second slope
SF of small firm	2.1116(2)*	1.4334(1)	2.9157(2)
SF of large firm	1.6595(2)*	1.0926(0.33)	2.4996(0.67)
Market structure: one large firm and two small firms			
	intercept	the first slope	the second slope
SF of small firms	1.9928(2)	1.6480(1.25)	3.2272(2.5)
SF of large firm	1.6350(2)	1.0492(0.4167)	1.8976(0.834)
structure: one large firm and three small firms			
	intercept	the first slope	the second slope
SF of small firms	1.9680(2)	1.8735(1.5)	3.5981(3)
SF of large firm	1.6129(2)	1.0808(0.5)	1.6230(1)

* intercepts and slopes of MC curves are in parentheses

We next calculate the expected quantities and expected prices with these equilibrium supply functions. The expected price decreases as the number of firms rises and increases as the asymmetry between firm sizes grows. The expected quantity increases as the number of firms rises and decreases as the asymmetry between firm sizes grows.

Table 8. Expected price and quantity with supply function equilibrium

market structure	number of firms	expected quantity	expected price
symmetric firms	2	12.6357	9.4486
symmetric firms	3	13.7758	8.3085
symmetric firms	4	14.2572	7.8271
symmetric firms	5	14.5132	7.5711
symmetric firms	10	14.9530	7.1313
twice large	2	12.3693	9.7151
twice large	3	13.5659	8.5184
twice large	4	14.1113	7.9730
three times large	2	11.7898	10.2945
three times large	3	13.1485	8.9359
three times large	4	13.8451	8.2392

Fourth, to compare the market powers of various market structures, Lerner indexes[3] are calculated based on the previous simulation results. For purposes of comparison, two additional benchmark cases are also considered. First, on the assumption that electricity is storable, the Cournot equilibrium is calculated using the uniform distribution for the demand uncertainty. The power system is a prototype of a just-in-time-manufacturing systems in which all output must be consumed exactly at the time it is manufactured. If electricity is storable, firms produce electricity in off-peak times in an amount that is greater than immediate demand and save the remaining amount and to meet demand at peak times. In this case, the problem of firms is the same as Cournot competition, and firms aggregate all variant demands and optimize their production by maximizing their expected profits. Table 9 shows the results for Cournot quantities and Cournot

Table 9. Price and quantity of the Cournot equilibrium with storable goods

market structure	number of firms	Cournot quantity	Cournot price
symmetric firms	2	11.4768	10.6075
symmetric firms	3	12.6849	9.3994
symmetric firms	4	13.3896	8.6947
symmetric firms	5	13.8515	8.2328
symmetric firms	10	14.8770	7.2073
twice large	2	11.3470	10.7373
twice large	3	12.5482	9.5361
twice large	4	13.2612	8.8231
three times large	2	11.0439	11.0404
three times large	3	12.2059	9.8784
three times large	4	12.9373	9.1470

3) The Lerner index, named after the economist Abba Lerner, describes a monopoly's market power. Mathematically, it is measured with the following formula: $L=(P-MC)/P$, where L is the Lerner index, P is the selling price and MC is the marginal cost. For a perfectly competitive firm (where $P=MC$), $L=0$. It has no market power.

prices. The pattern of expected prices and expected quantities is the same as in the case of supply function equilibrium. The expected price decreases when the number of firms rises and increases as firms become more asymmetrical. The expected quantity increases as the number of firms rises and decreases as firms become more asymmetrical.

Next, the Cournot points for every demand realization are calculated, with the results shown in Table 10. If firms can adjust their production after demand is realized, then firms will produce at the point of the Cournot equilibrium for each demand. Because firms must submit one supply function for all variant demands, firms cannot attain Cournot points of variant demands.

Table 10. Expected price and quantity of the Cournot points

market structure	number of firms	expected quantity	expected price
symmetric firms	2	11.3092	10.7750
symmetric firms	3	12.3993	9.6851
symmetric firms	4	13.4229	8.6613
symmetric firms	5	13.4239	8.6605
symmetric firms	10	14.2943	7.7899
twice large	2	11.1320	10.9522
twice large	3	12.2297	9.8545
twice large	4	12.8699	9.2144
three times large	2	10.8912	11.1931
three times large	3	11.9680	10.1163
three times large	4	12.6392	9.4451

Table 11, compares the market powers of the supply function equilibrium obtained from previous simulations to the Cournot equilibrium for storable goods and Cournot points in various market structures. The means of the Lerner indexes of the supply functions are far smaller than those of the Cournot points. The means of Lerner indexes of the Cournot points are far smaller than those of the

Cournot equilibrium, assuming that electricity is storable. These difference stems from the production of electricity being a 'just-in-time-manufacturing system'.

Table 11. Lerner indexes

Lerner index of the market composed of symmetric firms			
number of firms	supply function Eq	Cournot Eq**	Cournot points
2	0.3585*(0.1169)	0.5410**	0.4795*(0.1234)
3	0.2312 (0.0850)	0.4498	0.3906(0.1030)
4	0.1674 (0.0648)	0.3850	0.3292(0.0883)
5	0.1305 (0.0519)	0.3365	0.2844(0.0770)
10	0.0614 (0.0255)	0.2065	0.1712(0.0490)
Lerner index of the market composed of asymmetric firms when the large firm is twice the size of small firms			
number of firms	supply function Eq	Cournot Eq **	Cournot points
2	0.3834*	0.5495	0.4912*
3	0.2568	0.4613	0.4051
4	0.1874	0.3976	0.3444
Lerner index of the market composed of asymmetric firms when the large firm is three times the size of small firms			
number of firms	supply function Eq	Cournot Eq **	Cournot points
2	0.4293*	0.5688	0.5066*
3	0.3021	0.4886	0.4264
4	0.2221	0.4278	0.3663

* Lerner indexes are means for supply function and Cournot points for each demand realization
** Cournot equilibrium is calculated assuming storable goods

The Lerner index decreases as the number of firms increases, and it increases as firm size becomes more asymmetrical. For example, the Lerner index of the market composed of three symmetric firms is almost the same as that of the market in which there are four firms, but one large firm is three times the size of the three small firms.

So far, the Lerner indexes have been calculated at the market level. Table 12, however, shows market power at the firm level. In an asymmetric market structure, the Lerner index of a large firm is larger than that of small firms.

Because the unit price paid is the same for all production supplied by firms in the uniform price auction, the proportion of the production of a small firm is larger than the proportion of the size of the firm. Because price is the same for every firms, the MC of the large firm is smaller. Large firms produce less in proportion to the size. The difference between the Lerner indexes of a large firm and those of small firms increases with the asymmetry of firm sizes.

Table 12. The Lerner index of each firms in the market with symmetric and asymmetric firms

	Lerner indexes
symmetric firms: 2 firms	(0.3585, 0.3585)
symmetric firms: 3 firms	(0.2312, 0.2312, 0.2312)
symmetric firms: 4 firms	(0.1674, 0.1674, 0.1674, 0.1674)
large firm is 2 times larger: 2 firms	(0.2700,0.4343)
large firm is 2 times larger: 3 firms	(0.1842,0.1842, 0.3246)
large firm is 2 times larger: 4 firms	(0.1390, 0.1390, 0.1390, 0.2563)
large firm is 3 times larger: 2 firms	(0.2329 0.4856)
large firm is 3 times larger: 3 firms	(0.1611 0.1611 0.3869)
large firm is 3 times larger: 4 firms	(0.1210, 0.1210, 0.1210, 0.3150)

Table 13 shows consumer surplus, profits and deadweight loss. Deadweight loss is calculated from a comparison with competitive equilibrium satisfying p = MC. Consumer surplus depends on equilibrium quantities and prices. As expected, consumer surplus increases as the number of firms of the market rises, and total profits decrease as the number of firms increases. Deadweight loss is larger in a less competitive market so that it decreases as the number of firms rises.

Table 13. Welfare analysis of markets composed of different numbers of symmetric firms

number of firms	consumer surplus	profit	deadweightloss
2	103.7535	90.5738	6.8982
3	123.1216	84.0705	2.3382
4	131.9601	76.4573	1.1130
5	136.8241	72.0659	0.6405
10	145.4539	63.9480	0.1285

Table 14. Comparison of consumer surplus

#	symmetric firms	twice as large as*	3 times as large as **
2	103.7535	99.7138	89.8344
3	123.1216	119.5683	111.9763
4	131.9601	129.3343	124.5264

* large firm is twice as large as small firms
** large firm is 3 times as large as small firms

Total profits of symmetric firms are smaller than those of the same number of asymmetric firms. This paper intentionally structures the market so that production units of a large firm are obtained by multiplying the production units of a small firm. The results indicate that the profit of a large firm is smaller than the proportionate profit of a smaller firm. In asymmetric market structures, small firms tend to bid more aggressively, and consequently, the amount of demand a small firm serves is large in proportion to its size. Because the unit price paid is the same for all production supplied by firms in the uniform price auction, a small firm's proportionate profit is larger than the proportionate size of the firm.

Table 15. Profits

	profits of firms
symmetric firms: 2	(45.2869, 45.2869)
symmetric firms: 3	(28.0235, 28.0235, 28.0235)
symmetric firms: 4	(19.1143, 19.1143, 19.1143, 19.1143)
large firm is 2 times larger: 2 firms	(38.5848, 61.7516)
large firm is 2 times larger: 3 firms	(22.9364, 22.9363, 40.3368)
large firm is 2 times larger: 4 firms	(16.1696, 16.1695, 16.1695, 29.8040)
large firm is 3 times larger: 2 firms	(33.7527, 71.1358)
large firm is 3 times larger: 3 firms	(20.7014, 20.7014, 49.4103)
large firm is 3 times larger: 4 firms	(14.6063, 14.6063, 14.6063, 37.7091)

Table 16 compares the deadweight loss in different market structures. As expected, dead-weight loss is larger when the market consists of a smaller number of firms. Severe asymmetry between firms results in large deadweight loss. For example, the deadweight loss in the market which consists of two small firms and one large firm 3 times as large as each of the small firms is almost the same as the market composed of two symmetric firms.

Table 16. Comparison of deadweight loss

#	symmetric firms	large firm is 2 times larger	large firm is 3 times larger
2	6.8982	9.4801	14.8074
3	2.3382	3.7525	6.7408
4	1.1130	1.8834	3.4759

Even though asymmetric equilibria for symmetric market structures are not excluded in the computation process finding the optimized supply function equilibrium, only symmetric equilibria are obtained. Symmetric firms submit the same bid functions so that production facilities are operated in equilibrium up to the same marginal cost. However, asymmetric firms submit different bid functions, and as a result, the higher cost production facility of one firm is operated rather than the lower cost production facility(ies) of the other firm(s). Table 17 shows the efficiency loss from the operation of less efficient production facilities. A more severe asymmetry results in a larger efficiency loss.

Table 17. Comparison of efficiency loss

#	symmetric firms	large firm is 2 times larger	large firm is 3 times larger
2	0	1.2761	2.6407
3	0	0.7920	2.1115
4	0	0.4625	1.3744

Ⅴ. Conclusion

This paper models the restructured wholesale electricity markets as oligopolies facing uncertain demand and "capacity constraints" where each firm chooses as its strategy a "supply function", as described by Klemperer and Meyer. It intends to provide comparative statics for the SFE model, which incorporates the more realistic market structure features. To avoid the difficulties of solving the SFE model analytically, especially considering the more realistic features of the electricity market structure and to facilitate the computation of optimal strategies, this paper considers piecewise linear bid(or supply) functions, imposing restrictions that allow for a limited number of knots. Simulations using various combinations of firms are implemented to compare the performance of various market structures. The results of these simulation show that firms' equilibrium supply functions are steeper with fewer firms and with a more severe asymmetry between the small firms and a large firm. We also compare the Cournot equilibrium, assuming storability, and the Cournot points of every demand realization. The results show that the Lerner index of supply function equilibrium with capacity constraints is far smaller than that of the Cournot equilibrium, assuming storability, and the Cournot points. The Lerner index decreases as the number of firms rises and it increases as the asymmetry between firm sizes grows. Because we can get the similar results with Cournot equilibrium, these comparative statics of SPE accord well with our predictions. The results showing that the firms' equilibrium supply function are steeper and the Lerner indexes are larger with a smaller variance of demand correlate to the characteristic of the supply function equilibrium where each agent's best strategy considers all demand uncertainty or variation. From the supplier's point of view, they can better optimize their strategies and earn greater profits with a smaller variance of demand.

We can infer from these simulation results that the more firms in the restructured wholesale electricity market should result in lower price and smaller deadweight loss. A market composed of similarly scaled firms should have a lower market price and smaller deadweight loss than a market in which one firm dominates the other firms in scale. However, firms in a power exchange market in which they submit price-quantity schedules cannot attain Cournot profits. Because the firm's best strategies must consider the variance of demand, its profits are limited.

This paper's modeling framework and simulation method can serve for the study of various policy issues, including investigating the effects of market design and alternative bidding rules on expected price, price variability, and economic efficiency. In addition to this modeling framework and simulation method, more meaningful results may be produced using more realistic cost and demand functions from real electricity market data.

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초 록

생산 능력 제약하 공급 함수 균형: 모형과 시뮬레이션

염수현*

본 논문은 구조개혁된 전력 도매시장을 각각의 기업들이 불확실한 수요와 생산능력 제약에 직면하고 있으며 Klemperer와 Meyer에 의해 도입된 공급 함수를 전략으로 취하고 있는 시장으로 모형화 하였다. 그리고 기업들의 최적 전략을 계산하는 것을 용이하게 하기 위하여 하나의 결절점을 허용하면서 구간별로 선형인 입찰(공급)함수만을 고려하였다. 본 논문은 기업들의 균형 전략을 찾을 수 있는 알고리듬을 제시하고 시뮬레이션을 수행하였다. 이러한 작업은 기업의 수, 기업들 규모의 차이, 수요의 변동성과 같은 요소들이 전력 거래소(Electricity Exchange)의 성과에 어떠한 영향을 미치는 지를 평가할 수 있게 한다. 시뮬레이션은 다양한 기업들의 조합으로 구성되었는데, 시뮬레이션 결과는 기업수가 적을수록, 수요의 분산이 적을수록, 큰 기업과 작은 기업들 간 규모차가 클수록 기업들의 공급함수가 가파르게 됨을 보여준다. 또한 공급함수를 전략으로 취하는 공급함수 균형(Supply Function Equilibria)을 실현되는 각각의 수요에 대한 Cournot 균형점들, 그리고 전력이 저장될 수 있는 상품이라는 가정하에 계산된 Cournot 균형과 비교하였다. 비교 결과 생산능력 제약하의 공급함수 균형(Supply Function Equilibrium)의 러너 인덱스(Lerner Index)는 전력이 저장될 수 있는 상품이라는 가정 하에 계산된 Cournot 균형 및 실현되는 각각의 수요에 대한 Cournot 균형점들에 비해 훨씬 작았다. 러너 인덱스는 기업의 수가 증가할수록 감소하고 기업 규모들 간 차이가 클수록 증가하였다.

주요 단어 : 공급함수 균형, 생산능력 제약, 전력 산업
경제학문헌목록 주제분류 : L1, Q3

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